Soluciones propuesta A

1. a) $a_6 = 19$, $a_7 = 33$, $a_8 = 25$, $a_n = 3n + 1$

b) $a_6 = \frac{11}{12}$, $a_7 = \frac{13}{14}$, $a_8 = \frac{15}{16}$, $a_n = \frac{2n-1}{2n}$

c) $a_6 = 36$, $a_7 = 49$, $a_8 = 64$, $a_n = n^2$

d) $a_6 = 37$, $a7 = 50$, $a_8 = 65$, $a_n = n^2 + 1$

2. a) $a_0 = 2,\ a_1 = 2^{\frac{1}{2}},\ a_2 = 2^{\frac{1}{4}},$

$a_3 = 2^{\frac{1}{8}},\ a_4 = 2^{\frac{1}{16}},\ a_5 = 2^{\frac{1}{32}}$

b) $a_n = 2^{\frac{1}{2^n}}$

c) $\lim_{n\to\infty} 2^{\frac{1}{2^n}} = k \Rightarrow \log_2 k = \lim_{n\to\infty} \frac{1}{2^n} = 0 \Rightarrow k = 1$

El límite de la sucesión es 1.

d) Está acotada superiormente por $a_0 = 2$ e inferiormente por su límite, 1.

3. a) $a_1 = 5,\quad a_2 = 4,\quad a_3 = \frac{11}{3}$

$3 + \frac{2}{s} = \frac{28}{9} \Leftrightarrow \frac{2}{s} = \frac{1}{9} \Leftrightarrow s = 18$

b) $a_{n+1} - a_n = \left(3 + \frac{2}{n+1}\right) - \left(3 + \frac{2}{n}\right) = \frac{-2}{(n+1)n} < 0$

La sucesión es estrictamente decreciente.

c) $\lim a_n = \lim\left(3 + \frac{2}{n}\right) = 3 + 0 = 3$

$|a_n - 3| < \varepsilon;\ \left|3 + \frac{2}{n} - 3\right| < 0{,}001 \Leftrightarrow \left|\frac{2}{n}\right| < 0{,}001$

$2 < 0{,}001n \Rightarrow n > 2000$. A partir de a_{2000}.

4. a) $l = \lim_{n\to\infty} \frac{-2n^2 - 4n}{n^2 - 1} = -2$

b) $l = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

c) $l = \lim_{n\to\infty} \frac{-n^2}{3n} = -\infty$

d) $l = \lim_{n\to\infty} \frac{3n}{\sqrt{n^2+n} + \sqrt{n^2-2n}} = \frac{3}{\sqrt{1}+\sqrt{1}} = \frac{3}{2}$

e) $l = \lim_{n\to\infty} \left(\frac{3n+1}{2n+1}\right)^{n+2} = \left(\frac{3}{2}\right)^{+\infty} = +\infty$

f) $l = \lim_{n\to\infty} \frac{\frac{1+n}{2}n}{2n^2} = \frac{1}{4}$

5. a) $\lim_{x\to -3^-}\left(\frac{x+2}{x^2-9}\right) = -\infty$, $\lim_{x\to -3^+}\left(\frac{x+2}{x^2-9}\right) = +\infty$

$\lim_{x\to 3^-}\left(\frac{x+2}{x^2-9}\right) = -\infty$, $\lim_{x\to 3^+}\left(\frac{x+2}{x^2-9}\right) = +\infty$

b) $\lim_{x\to 1^-}\left(\frac{x^2-3x}{x^2-x}\right) = +\infty$, $\lim_{x\to 1^+}\left(\frac{x^2-3x}{x^2-x}\right) = -\infty$

$\lim_{x\to 0}\left(\frac{x^2-3x}{x^2-x}\right) = \lim_{x\to 0}\left(\frac{x(x-3)}{x(x-1)}\right) = \lim_{x\to 0}\frac{x-3}{x-1} = 3$

6. a) –1

b) –20

c) +∞

d) –∞

e) –∞

f) +∞

7. Estos límites son del tipo $\frac{0}{0}$:

a) $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1}\frac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{(x-1)\left(\sqrt{x}+1\right)} =$

$= \lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x}+1\right)} = \lim_{x\to 1}\frac{1}{\sqrt{x}+1} = \frac{1}{2}$

b) $\lim_{x\to 4}\frac{\sqrt{x}-2}{\sqrt{12+x}-4} = \lim_{x\to 4}\frac{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)\left(\sqrt{12+x}+4\right)}{\left(\sqrt{12+x}-4\right)\left(\sqrt{12+x}+4\right)\left(\sqrt{x}+2\right)} =$

$= \lim_{x\to 4}\frac{(x-4)\left(\sqrt{12+x}+4\right)}{(x-4)\left(\sqrt{x}+2\right)} = \lim_{x\to 4}\frac{\left(\sqrt{12+x}+4\right)}{\left(\sqrt{x}+2\right)} = 2$

8. $\lim_{x\to -1^-}\left(\frac{x}{|x|-1}\right) = -\infty$; $\lim_{x\to -1^+}\left(\frac{x}{|x|-1}\right) = +\infty$

$\lim_{x\to 1^-}\left(\frac{x}{|x|-1}\right) = -\infty$; $\lim_{x\to 1^+}\left(\frac{x}{|x|-1}\right) = +\infty$

$\lim_{x\to -\infty}\left(\frac{x}{|x|-1}\right) = \lim_{x\to -\infty}\left(\frac{x}{-x-1}\right) = -1$

$\lim_{x\to +\infty}\left(\frac{x}{|x|-1}\right) = \lim_{x\to +\infty}\left(\frac{x}{+x-1}\right) = 1$

9. a) $\lim_{x\to -1}\frac{x^2+3x+2}{x^3+x^2+x+1} = \lim_{x\to -1}\frac{(x+1)(x+2)}{(x+1)(x^2+1)} = \frac{1}{2}$

b) $\lim_{x\to 2}\frac{(x-2)(x^3-x^2-2x)}{(x-2)(x+2)(x^2+4)} = \frac{0}{32} = 0$

Soluciones propuesta B

1. a) $a_6 = -5$, $a_7 = -8$, $a_8 = -11$, $a_n = 13 - 3n$

b) $a_6 = \frac{12}{13}$, $a_7 = \frac{14}{15}$, $a_8 = \frac{16}{17}$, $a_n = \frac{2n}{2n+1}$

c) $a_6 = 216$, $a_7 = 343$, $a_8 = 512$, $a_n = n^3$

d) $a_6 = 215$, $a_7 = 342$, $a_8 = 511$, $a_n = n^3-1$

2. a) $a_0 = 1$, $a_1 = -\frac{1}{3}$, $a_2 = \frac{1}{3^2}$, $a_3 = -\frac{1}{3^3}$,

$a_4 = \frac{1}{3^4}$, $a_5 = -\frac{1}{3^5}$

b) $a_n = (-1)^n \frac{1}{3^n}$

c) $b_n = \frac{1}{3^n}$, $c_n = -\frac{1}{3^n}$

$$\frac{b_{n+1}}{b_n} = \frac{\frac{1}{3^{n+1}}}{\frac{1}{3^n}} = \frac{1}{3}, \quad \frac{c_{n+1}}{c_n} = \frac{-\frac{1}{3^{n+1}}}{-\frac{1}{3^n}} = \frac{1}{3}$$

$$\frac{a_{n+1}}{a_n} = \frac{(-1)^{n+1}\frac{1}{3^{n+1}}}{(-1)^n \frac{1}{3^n}} = -\frac{1}{3}$$

b_n y c_n son progresiones geométricas de razón $r = \frac{1}{3}$. a_n es una progresión geométrica de razón $r = -\frac{1}{3}$.

d) Como se puede ver en los primeros términos, a_n es oscilante.

e) Superior: $a_0 = 1$. Inferior: $a_1 = -\frac{1}{3}$.

f) $c_n \le a_n \le b_n \Rightarrow \lim\limits_{n\to\infty} c_n \le \lim\limits_{n\to\infty} a_n \le \lim\limits_{n\to\infty} b_n \Rightarrow$

$$\Rightarrow \lim_{n\to\infty}\left(-\frac{1}{3^n}\right) \le \lim_{n\to\infty} a_n \le \lim_{n\to\infty}\frac{1}{3^n} \Rightarrow$$

$$\Rightarrow 0 \le \lim_{n\to\infty} a_n \le 0 \Rightarrow \lim_{n\to\infty} a_n = 0$$

3. a) $$\lim_{n\to\infty}\left(\frac{n^2}{n-1} - \frac{n^2+1}{n-2}\right) = \lim_{n\to\infty}\left(\frac{-n^2-n+1}{n^2-3n+2}\right) = -1$$

b) $$\lim_{n\to\infty}\left(\frac{2n^2}{n+1}\cdot\frac{n+4}{5n^2}\right) = \lim_{n\to\infty}\frac{2n^3+8n^2}{5n^3+5n^2} = \frac{2}{5}$$

c) $$\lim_{n\to\infty}\left(\sqrt{2n^2+1} - \sqrt{n^2+1}\right) =$$

$$= \lim_{n\to\infty}\frac{n^2}{\sqrt{2n^2+1}+\sqrt{n^2+1}} = +\infty$$

d) $$\lim_{n\to\infty}\left(\frac{2n^2}{3n+1}\right)^{\frac{3n^2+2}{5n-3}} = e^{\lim\limits_{n\to\infty}\left(\frac{3n^2+2}{5n-3}\right)\left(\frac{2n^2}{3n+1}\right)} = \left(e^{+\infty}\right) = +\infty$$

e) $$\lim_{n\to\infty}\sqrt[3]{\frac{n+3}{3-8n}} = \sqrt[3]{\frac{1}{-8}} = -\frac{1}{2}$$

f) $$\lim_{n\to\infty}\frac{2+4+6+...+2n}{n^2+1} = \lim_{n\to\infty}\frac{\frac{2+2n}{2}n}{n^2+1} = 1$$

4. a) 4 b) –3 c) +∞ d) –∞ e) +∞ f) +∞

5. Todos estos límites son del tipo $\frac{\infty}{\infty}$.

a) Simplificando por x^2:

$$\lim_{x\to+\infty}\frac{x^2-x+1}{x^2+3} = \lim_{x\to+\infty}\frac{x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)}{x^2\left(1+\frac{3}{x^2}\right)} = 1$$

b) Simplificando por x:

$$\lim_{x\to-\infty}\frac{x^2+4}{x+1} = \lim_{x\to-\infty}\frac{x+\frac{4}{x}}{1+\frac{1}{x}} = -\infty$$

6. a) $$\lim_{x\to-3^-}\left(\frac{x-3}{x+3}\right) = +\infty, \quad \lim_{x\to-3^+}\left(\frac{x-3}{x+3}\right) = -\infty$$

b) $$\lim_{x\to3^-}\frac{1}{(x-3)^2} = +\infty, \quad \lim_{x\to3^+}\frac{1}{(x-3)^2} = +\infty$$

c) $$\lim_{x\to1^-}\left(\frac{x^4+3x}{x^2-x}\right) = -\infty \quad \lim_{x\to1^+}\left(\frac{x^4+3x}{x^2-x}\right) = +\infty$$

$$\lim_{x\to0}\left(\frac{x^4+3x}{x^2-x}\right) = \lim_{x\to0}\frac{x\left(x^3+3\right)}{x(x-1)} = \frac{3}{-1} = -3$$

7. a) $$\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-4}-x\right)\left(\sqrt{x^2-4}+x\right)}{\left(\sqrt{x^2-4}+x\right)} =$$

$$= \lim_{x\to+\infty}\frac{-4}{\sqrt{x^2-4}+x} = 0$$

b) $$\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\left(\sqrt{x^2+x}+x\right)} =$$

$$= \lim_{x\to+\infty}\frac{x}{\sqrt{x^2+x}+x} = \lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{2}$$

8. $$\lim_{x\to-1^-}\left(\frac{x^2-1}{|x|-1}\right) = \lim_{x\to-1^-}\frac{(x+1)(x-1)}{-x-1} = \lim_{x\to-1^-}\frac{x-1}{-1} = 2$$

$$\lim_{x\to-1^+}\left(\frac{x^2-1}{|x|-1}\right) = \lim_{x\to-1^+}\frac{x-1}{-1} = 2, \quad \lim_{x\to1}\left(\frac{x^2-1}{|x|-1}\right) = 2$$

$$\lim_{x\to-\infty}\left(\frac{x^2-1}{|x|-1}\right) = +\infty, \quad \lim_{x\to+\infty}\left(\frac{x^2-1}{|x|-1}\right) = +\infty$$