Soluciones

1. La función buscada es $f(x) = 2x^3 - 2x^2 + 5x + K$

Para determinar la constante K, se exige que $f(2) = 25$, es decir, $16 - 8 + 10 + K = 25 \Rightarrow K = 7 \Rightarrow$
$\Rightarrow f(x) = 2x^3 - 2x^2 + 5x + 7 \Rightarrow f(0) = 7$.

2. $f''(x) = 6x + 6 \Rightarrow f'(x) = 3x^2 + 6x + C$. Como hay un máximo relativo en $x = -3$, entonces $f'(-3) = 0 \Rightarrow$
$\Rightarrow C = -9$ y la derivada es $f'(x) = 3x^2 + 6x - 9$.

$f(x) = x^3 + 3x^2 - 9x + K$ y como $f(-3) = 17 \Rightarrow K = -10$

Por tanto, $f(x) = x^3 + 3x^2 - 9x - 10$.

Inflexión: $6x + 6 = 0 \Rightarrow x = -1 \Rightarrow I(-1, 1)$

Mínimo:

$$3x^2 + 6x - 9 = 0 \Rightarrow \begin{cases} x_1 = -3 \Rightarrow \text{máximo } M(-3, 17) \\ x_2 = 1 \Rightarrow \text{mínimo } m(1, -15) \end{cases}$$

Punto de corte con Y: $P(0, -10)$

3. $v = \int a\,dt = \int -10\,dt = -10t + C$, $v(2) = 12 \Rightarrow C = 32$

$s = \int v\,dt = \int (-10t + 32)\,dt = -5t^2 + 32t + K$

$s(2) = 48 \Rightarrow -20 + 64 + K = 48 \Rightarrow K = 4$

a) $v(t) = -10t + 32 \text{ m s}^{-1}$ d) $s_0 = s(0) = 4$ m

b) $v_0 = v(0) = 32 \text{ m s}^{-1}$ e) $s(4) = 52$ m, $v(4) = -8 \text{ m s}^{-1}$

c) $s(t) = -5t^2 + 32t + 4$ m

Se puede interpretar como un lanzamiento vertical desde 4 m de altura con velocidad inicial 32 m s^{-1}. A los 4 s el móvil ya está bajando.

4. a) Tomando $\begin{cases} u = (\ln x)^2 \Rightarrow du = \dfrac{2}{x}\ln x\,dx \\ dv = dx \Rightarrow v = x \end{cases}$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \ln x\,dx = x(\ln x)^2 + 2x(1 - \ln x) + k$$

Ya que la integral de $\ln x$ se hace por partes:

$$\int \ln x\,dx = x\ln x - x = -x(1 - \ln x)$$

b) Tomando $\begin{cases} u = x \Rightarrow du = dx \\ dv = \dfrac{dx}{\cos^2 x} \Rightarrow v = \operatorname{tg} x \end{cases}$

$$\int \frac{x}{\cos^2 x}dx = x\operatorname{tg} x - \int \operatorname{tg} x\,dx = x\operatorname{tg} x + \ln|\cos x| + K$$

5. a) Tomando $\begin{cases} u = \operatorname{arctg} x \Rightarrow du = \dfrac{dx}{x^2+1} \\ dv = 2x\,dx \Rightarrow v = x^2 \end{cases}$

$$\int 2x \cdot \operatorname{arctg} x \cdot dx = x^2 \operatorname{arctg} x - \int \frac{x^2 + (1-1)}{x^2+1}dx =$$

$$= x^2\operatorname{arctg} x - \int 1\,dx + \int \frac{1}{x^2+1}dx = (x^2+1)\operatorname{arctg} x - x + K$$

b) $\int (3x+1) \cdot e^x \cdot dx = (3x+1) \cdot e^x - 3\int e^x\,dx =$
$= (3x + 1) \cdot e^x - 3e^x + C = (3x - 2) \cdot e^x + C$

6. a) $I = \int e^{2x+1} \cdot \cos(x-2) \cdot dx =$

$= e^{2x+1}\operatorname{sen}(x-2) + 2e^{2x+1}\cos(x-2) - 4I \Rightarrow$

$\Rightarrow I = \dfrac{1}{5}e^{2x+1}(\operatorname{sen}(x-2) + 2\cos(x-2)) + K$

b) Tomando $\begin{cases} u = \operatorname{sen}^3 x \Rightarrow du = 3\operatorname{sen}^2 x\cos x\,dx \\ dv = \operatorname{sen} x\,dx \Rightarrow v = -\cos x \end{cases}$

$$\int \operatorname{sen}^4 x\,dx = -\operatorname{sen}^3 x\cos x + 3\int \operatorname{sen}^2 x\cos^2 x\,dx$$

$$I = -\operatorname{sen}^3 x\cos x + 3\int \operatorname{sen}^2 x\,dx - 3I \Rightarrow$$

$$I = \frac{1}{4}\left(-\operatorname{sen}^3 x\cos x + \frac{3}{2}(x - \operatorname{sen} x\cos x)\right) + K$$

7. $$\int\left(\frac{x^2 - 3x + 5}{x}\right)dx = \int\left(x - 3 + \frac{5}{x}\right)dx = \frac{1}{2}x^2 - 3x + 5\ln|x| + K$$

8. a) $\displaystyle\int \frac{5}{x^2 - 3x - 4}dx = -\ln|x+1| + \ln|x-4| + C$, ya que

$$\frac{5}{x^2 - 3x - 4} = \frac{A}{x+1} + \frac{B}{x-4} \Rightarrow A(x-4) + B(x+1) = 5$$

de donde $A = -1$ y $B = 1$.

b) $\displaystyle\int \frac{x^4 - 2x^3 + x^2 - 5x + 6}{x^3 - 3x^2}dx$ se descompone:

$$\int\left[(x+1) + \frac{3}{x-3} + \frac{1}{x} - \frac{2}{x^2}\right]dx =$$

$$= \frac{1}{2}x^2 + x + 3\ln|x-3| + \ln|x| + \frac{2}{x} + K$$

9. a) $$\int \frac{x+1}{x^2 + 6x + 10}dx = \frac{1}{2}\int \frac{2x + 6 - 4}{x^2 + 6x + 10}dx \overset{y = x+3}{=}$$

$$= \frac{1}{2}\int \frac{2x+6}{x^2 + 6x + 10}dx - \int \frac{2dy}{y^2 + 1} =$$

$$= \frac{1}{2}\ln(x^2 + 6x + 10) - 2\operatorname{arctg}(x+3) + K$$

b) $$\int \frac{5}{x^4 - 1}dx = \int\left[\frac{\frac{5}{4}}{x-1} + \frac{-\frac{5}{4}}{x+1} + \frac{-\frac{5}{2}}{x^2+1}\right]dx =$$

$$= \frac{5}{4}\ln|x-1| - \frac{5}{4}\ln|x+1| - \frac{5}{2}\operatorname{arctg} x + K$$

10. a) $$\int \frac{1}{1 - \cos x}dx = \int \frac{1 + \cos x}{\operatorname{sen}^2 x}dx =$$

$$= \int \frac{1}{\operatorname{sen}^2 x}dx + \int \frac{\cos x}{\operatorname{sen}^2 x}dx = -\operatorname{cotg} x - \operatorname{cosec} x + K$$

b) $$\int \frac{x - 4\sqrt{x}}{\sqrt[3]{x}}dx = \int\left(x^{\frac{2}{3}} - 4x^{\frac{1}{6}}\right)dx = \frac{3}{5}x^{\frac{5}{3}} - \frac{24}{7}x^{\frac{7}{6}} + K$$

11. a) $$\int \operatorname{sen}^3 x \cdot dx = \int (1 - \cos^2 x)\operatorname{sen} x\,dx \overset{t = \cos x}{=}$$

$$= \int (1 - t^2)\cdot(-dt) = -t + \frac{1}{3}t^3 + K = -\cos x + \frac{1}{3}\cos^3 x + K$$

b) $$\int \frac{1 + \operatorname{sen}^2 x}{\cos^4 x}dx = \int\left(\frac{1}{\cos^2 x} + \frac{\operatorname{sen}^2 x}{\cos^2 x}\right)\frac{1}{\cos^2 x}dx =$$

$$= \int (1 + \operatorname{tg}^2 x + \operatorname{tg}^2 x)\frac{1}{\cos^2 x}dx \overset{t = \operatorname{tg} x}{=}$$

$$= \int (1 + 2t^2)\,dt = t + \frac{2}{3}t^3 + K = \operatorname{tg} x + \frac{2}{3}\operatorname{tg}^3 x + K$$