1. Se divide el intervalo [2, 6] en n intervalos iguales de amplitud $\frac{4}{n}$ mediante la partición:

$$\left\{2,\ 2+\frac{4}{n},\ 2+2\left(\frac{4}{n}\right),\ 2+3\left(\frac{4}{n}\right),\ ...,\ 2+(n-1)\left(\frac{4}{n}\right),\ 6\right\}$$

$$\sum_1^n f(x_i)\cdot c_i = \frac{4}{n}\left[1+1+\frac{2}{n}+1+\frac{4}{n}+...+1+\frac{2(n-1)}{n}\right]=$$

$$=\frac{4}{n}\left[(1+1+1+...+1)+\frac{2+4+6+...+2(n-1)}{n}\right]=$$

$$=\frac{4}{n}\left[n+\frac{\frac{2+2(n-1)}{2}(n-1)}{n}\right]=\frac{4}{n}(n+n-1)=\frac{8n-4}{n}$$

Con los extremos superiores es análogo y se obtiene $\frac{8n+4}{n}$. En ambos casos el límite es igual:

$$\lim_{n\to\infty}\left(\frac{8n-4}{n}\right)=\lim_{n\to\infty}\left(\frac{8n+4}{n}\right)=8$$

2. Todos los intervalos tienen amplitud $c_i = 0{,}5$.

$$\sum_1^8 f(x_i)\cdot c_i = 0{,}5(1+1{,}8+...+3+3{,}2+3{,}5)=19{,}4$$

3. a) $\int_1^5 (2x+1)\,dx = [x^2+x]_1^5 = (25+5)-(1+1)=28$

b) $\int_{-1}^3 \frac{2}{x+2}\,dx = [2\ln|x+2|]_{-1}^3 = (2\ln 5 - 2\ln 1) = 2\ln 5$

c) $\int_0^{\frac{\pi}{2}} \cos x\,dx = [\operatorname{sen} x]_0^{\frac{\pi}{2}} = \operatorname{sen}\left(\frac{\pi}{2}\right) - \operatorname{sen} 0 = 1$

d) $\int_{-1}^1 e^{-x}dx = [-e^{-x}]_{-1}^1 = (-e^{-1})-(-e^1) = e - \frac{1}{e}$

4. a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow 3 = \frac{k}{1} \Rightarrow k = 3$

b) $\int_{-2}^e f(x)\,dx = \int_{-2}^1 (4-x^2)\,dx + \int_1^e \frac{3}{x}\,dx =$

$=\left[4x-\frac{x^3}{3}\right]_{-2}^1 + [3\ln x]_1^e = 9+3=12$

5. a) $F'(x) = x^2+4x+5$

b) $G(x) = \int_x^5 \ln t\,dt = -\int_5^x \ln t\,dt \Rightarrow G'(x) = -\ln x$

c) $H(x) = \int_{2x}^{x^2+3}\sqrt{t}\,dt = \int_{2x}^0 \sqrt{t}\,dt + \int_0^{x^2+3}\sqrt{t}\,dt =$

$= -\int_0^{2x}\sqrt{t}\,dt + \int_0^{x^2+3}\sqrt{t}\,dt \Rightarrow$

$\Rightarrow H'(x) = 2\sqrt{2x} + 2x\sqrt{x^2+3}$

6. $F'(x) = x^2-4x+3 = (x-1)(x-3)$. Se anula para $x=1$ y para $x=3$.

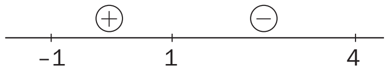
Se halla $F(0)=0$, $F(1) = \int_0^1 (t^2-4t+3)\,dt = \frac{4}{3}$,

$F(3) = \int_0^3 (t^2-4t+3)\,dt = 0$, $F(5) = \frac{20}{3}$

Máximo: $\frac{20}{3}$; mínimo: 0.

7. Raíces: $0 = x^3-4x^2-x+4 \Rightarrow \begin{cases} x = \pm 1 \\ x = 4\end{cases}$

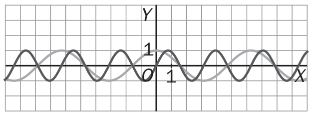
Signo:



$$S=\int_{-1}^1 (x^3-4x^2-x+4)\,dx + \int_1^4 (-x^3+4x^2+x-4)\,dx =$$

$$=\frac{253}{12}$$

8. Puntos de intersección:



$\operatorname{sen} 2x = \cos x \Rightarrow x = -\frac{\pi}{2},\ x=\frac{\pi}{6},\ x=\frac{\pi}{2}$

$$S_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (\cos x - \operatorname{sen} 2x)\,dx = \left[\operatorname{sen} x + \frac{\cos 2x}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}} = \frac{9}{4}$$

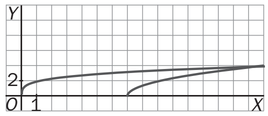
$$S_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\operatorname{sen} 2x - \cos x)\,dx = \left[-\operatorname{sen} x - \frac{\cos 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{4}$$

9. $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1 \Rightarrow y^2 = \frac{b^2}{a^2}(a^2-x^2)$

$$V = 2\pi\int_0^a y^2\,dx = 2\pi\frac{b^2}{a^2}\int_0^a (a^2-x^2)\,dx = \frac{4}{3}\pi ab^2$$

$$V_2 = 2\pi\int_0^b x^2\,dy = 2\pi\frac{a^2}{b^2}\int_0^b (b^2-y^2)\,dy = \frac{4}{3}\pi a^2 b$$

10. Punto de corte (16, 4).



$$V = \pi\left(\int_0^{16} 4\sqrt{x}\,dx - \int_7^{16}\frac{16}{9}(x-7)\,dx\right) = \frac{296}{3}\pi$$

11. $f(x) = \frac{2}{3}\sqrt{(x-1)^3} \Rightarrow f'(x) = \sqrt{x-1}$

$$L = \int_1^4 \sqrt{1+\left(\sqrt{x-1}\right)^2}\,dx = \int_1^4 \sqrt{x}dx = \frac{14}{3}\text{ u}$$

12. $y = e^x \Rightarrow y' = e^x \qquad L = \int_0^{\ln 3}\sqrt{1+e^{2x}}\,dx$

Con el cambio de variable:

$1+e^{2x} = t^2 \Rightarrow dx = \frac{t}{t^2-1}dt,\ \begin{cases} x = 0 \Rightarrow t = \sqrt{2} \\ x = \ln 3 \Rightarrow t = \sqrt{10}\end{cases}$

$$L = \int_{\sqrt{2}}^{\sqrt{10}} \frac{t^2}{t^2-1}\,dt = \left[t+\frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)\right]_{\sqrt{2}}^{\sqrt{10}} =$$

$$=\left(\sqrt{10}-\sqrt{2}\right)+\frac{1}{2}\left[\ln\left(\frac{\sqrt{10}-1}{\sqrt{10}+1}\right)-\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]$$

13. $dW = F\cdot dx \Rightarrow W_{1\to 2} = \int_{x_1}^{x_2} F(x)\cdot dx$

$$W = \int_{0{,}10}^{0{,}05} (-2000x)\,dx = [-1000x^2]_{0{,}10}^{0{,}05} = 7{,}5\ \text{J}$$