Soluciones

1. a) $A_L(x) = 4 \cdot (x \cdot 3x) = 12x^2$ cm^2 c) $V(x) = x^2 \cdot 3x = 3x^2$ cm^3

b) $A_T(x) = 2x^2 + 12x^2 + 14x^2$ cm^2 d) $A_L(5) = 12 \cdot 5^2 = 300$ cm^2, $A_T(5) = 14 \cdot 5^2 = 350$ cm^2, $V(5) = 3 \cdot 5^3 = 375$ cm^3

2. a) $f(56) = \dfrac{56}{2} = 28$ b) $f(101) = \dfrac{101+1}{2} = 51$ c) $(f \circ f \circ f)(422) = f(f(f(422))) = f(f(211)) = f(106) = 53$

d) La función f es no creciente, luego el mayor valor se obtiene para $x = 128$ y el menor para $x = 50$. Se calcula:

$(f \circ f \circ f \circ f \circ f \circ f \circ f)(128) = f(f(f(f(f(f(f(128))))))) = f(f(f(f(f(f(64)))))) = \dots = f(f(4)) = f(2) = 1$

$(f \circ f \circ f \circ f \circ f \circ f \circ f)(50) = f(f(f(f(f(f(f(50))))))) = f(f(f(f(f(f(25)))))) = f(f(f(f(f(13))))) = \dots = f(f(2)) = f(1) = 1$

Luego la función es $(f \circ f \circ f \circ f \circ f \circ f \circ f)(x) = 1$ constantemente en $50 \leqslant x \leqslant 128$.

3. a) $D(f) = \mathbf{R} = (-\infty, +\infty)$ c) $D(h) = \{x \in \mathbf{R} / 8 - 3x \geqslant 0\} = \left\{x \in \mathbf{R} / x \leqslant \dfrac{8}{3}\right\} = \left(-\infty, \dfrac{8}{3}\right]$

b) $D(g) = \{x \in \mathbf{R} / x^2 - x - 2 \neq 0\} = (-\infty, -1) \cup (-1, 2) \cup (2, +\infty)$ d) $D(k) = \{x \in \mathbf{R} / \operatorname{sen} x \geqslant 0\} = [2k\pi, (2k+1)\pi]\ \forall k \in \mathbf{Z}$

4. a) $(f+g)(-1) = f(-1) + g(-1) = (-3) + 1 = -2$ f) $(f + g)(x) = f(x) + g(x) = (x^2 - 4) + \dfrac{3}{x+4} = \dfrac{x^3 + 4x^2 - 4x - 13}{x+4}$

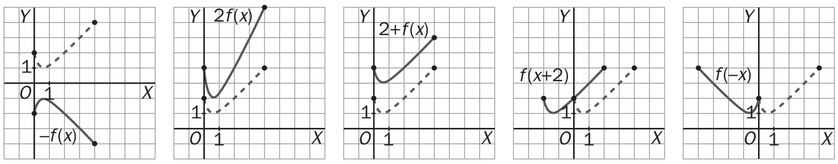
b) $(h \cdot f)(4) = h(4) \cdot f(4) = 2 \cdot 12 = 24$ g) $(g \cdot f)(x) = g(x) \cdot f(x) = \dfrac{3x^2 - 12}{x + 4}$

c) $(f \circ g)(2) = f(g(2)) = f\left(\dfrac{1}{2}\right) = \dfrac{1}{4} - 4 = -\dfrac{15}{4}$ h) $(g \circ f)(x) = g(f(x)) = g(x^2 - 4) = \dfrac{3}{x^2}$

d) $(g \circ h)(9) = g(h(9)) = g(3) = \dfrac{3}{7}$ i) $(f \circ h)(x) = f(h(x)) = f\left(\sqrt{x}\right) = x - 4$

e) $(h \circ f)(3) = \sqrt{5}$

5. Dominio: $D = [0, 4]$. Recorrido: $R = [1, 4]$.



6. Raíces: $x_1 = 1$, $x_2 = 5 \Rightarrow f(x) = a(x - 1)(x - 5)$. Pasa por $(0, 3) \Rightarrow 3 = 5a \Rightarrow a = \dfrac{3}{5}$. Por tanto, $f(x) = \dfrac{3}{5}x^2 - \dfrac{18}{5}x + 3$

7. a) 4 b) 2,4 c) 2,02 d) $\dfrac{f(1+h) - f(1)}{(1+h) - 1} = \dfrac{(1+h)^2 - 1}{h} = \dfrac{h^2 + 2h}{h} = 2 + h$ e) $\lim\limits_{h\to 0} \dfrac{f(1+h) - f(1)}{(1+h) - 1} = \lim\limits_{h\to 0}(2 + h) = 2$

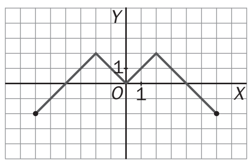
8. Dominio: $D = [-6, 4]$. Recorrido: $R = [-2, 2]$.

Máx. relativos: $M_1(-2, 2)$ y $M_2(2, 2)$.

Mín. relativos: $m_1(-6, -2)$, $m_2(0, 0)$ y $m_3(6, -2)$.

Crecimiento: $(-6, -2) \cup (0, 2)$.

Decrecimiento: $(-2, 0) \cup (2, 6)$.



9. a) $f(x) = \sqrt{x^2} = |x| \neq g(x) = x$ c) $f(x) = \log\left(\dfrac{x + 3}{2 - x}\right) = g(x) = \log(x + 3) - \log(2 - x)$

b) $f(x) = x^2 - x - 2 = g(x) = (x+1)(x-2)$ d) $f(x) = \sqrt{\dfrac{-2x - 10}{x - 2}} \neq g(x) = \dfrac{\sqrt{-2x - 10}}{\sqrt{x - 2}}$ pues $D(f) = [-5, 2) \neq D(g) = \varnothing$.

10. a) $+\infty$ b) 0, porque $0 < \dfrac{2}{3} < 1$ c) e d) $\sqrt{e}$

11. a) $\lim\limits_{x\to 1}\left(\dfrac{x^2 - 1}{x - 1}\right) = \lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{(x-1)} = \lim\limits_{x\to 1}(x + 1) = 2$ c) $\lim\limits_{x\to 3}\left(\dfrac{\sqrt{x} - \sqrt{3}}{x - 3}\right) = \lim\limits_{x\to 3}\dfrac{\sqrt{x} - \sqrt{3}}{\left(\sqrt{x} - \sqrt{3}\right)\left(\sqrt{x} + \sqrt{3}\right)} = \dfrac{1}{2\sqrt{3}}$

b) $\lim\limits_{x\to -2}\left(\dfrac{x^2 + 3x + 2}{x^2 - 4}\right) = \lim\limits_{x\to -2}\dfrac{(x+1)}{(x-2)} = \dfrac{1}{4}$ d) $\lim\limits_{x\to 0^+}\left(\dfrac{x^2 - 1}{x}\right) = \dfrac{-1}{0^+} = -\infty$

12. $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(x) \Rightarrow 8 = k - 10 \Rightarrow k = 18$