1. **Resuelve estos sistemas de ecuaciones:**

a) $\begin{cases} 2x - y - 1 = 0 \\ x^2 - 7 = y + 2 \end{cases}$ b) $\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = 1 - \dfrac{1}{xy} \\ xy = 6 \end{cases}$ c) $\begin{cases} x = 2y + 1 \\ \sqrt{x + y} - \sqrt{x - y} = 2 \end{cases}$

a) $\left.\begin{array}{l} y = 2x - 1 \\ y = x^2 - 9 \end{array}\right\}$

$x^2 - 9 = 2x - 1;\ \ x^2 - 2x - 8 = 0$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm 6}{2} = \begin{cases} 4 \\ -2 \end{cases}$$

$x_1 = 4;\ \ y_1 = 7$

$x_2 = -2;\ \ y_2 = -5$

b) $\left.\begin{array}{l} y + x = xy - 1 \\ xy = 6 \end{array}\right\}$

$y = 5 - x$

$x(5 - x) = 6;\ \ 5x - x^2 = 6;\ \ x^2 - 5x + 6 = 0 \begin{cases} x = 2 \\ x = 3 \end{cases}$

$x_1 = 2;\ \ y_1 = 3$

$x_2 = 3;\ \ y_2 = 2$

c) $x = 2y + 1$

$\sqrt{3y + 1} - \sqrt{y + 1} = 2;\ \ \sqrt{3y + 1} = 2 + \sqrt{y + 1}$

$3y + 1 = 4 + y + 1 + 4\sqrt{y + 1};\ \ 2y - 4 = 4\sqrt{y + 1};\ \ y - 2 = 2\sqrt{y + 1}$

$y^2 + 4 - 4y = 4y + 4;\ \ y^2 - 8y = 0$

$y = 8 \ \rightarrow\ x = 17$

$y = 0$ (no vale)

$x = 17;\ \ y = 8$

2. **Resuelve:**

$\begin{cases} x^2 + xy + y^2 = 21 \\ x + y = 1 \end{cases}$

$y = 1 - x;\ \ x^2 + x(1 - x) + (1 - x)^2 = 21$

$x^2 + x - x^2 + 1 + x^2 - 2x = 21;\ \ x^2 - x - 20 = 0$

$$x = \frac{1 \pm \sqrt{1 + 80}}{2} = \frac{1 \pm 9}{2} = \begin{cases} 5 \ \rightarrow\ y = -4 \\ -4 \ \rightarrow\ y = 5 \end{cases}$$

$x_1 = -4;\ \ y_1 = 5$

$x_2 = 5;\ \ y_2 = -4$

1. **Reconoce como escalonados y resuelve:**

a) $\begin{cases} x = 7 \\ 2x - 3y = 8 \\ 3x + y - z = 12 \end{cases}$

b) $\begin{cases} 3x + 4y = 0 \\ 2y = -6 \\ 5x + y - z = 17 \end{cases}$

c) $\begin{cases} 3x = -3 \\ 5y = 20 \\ 2x + y - z = -2 \end{cases}$

d) $\begin{cases} y = 4 \\ x - z = 11 \\ y - z = 7 \end{cases}$

a) $\left.\begin{array}{l} x = 7 \\ 2x - 3y = 8 \\ 3x + y - z = 12 \end{array}\right\}$ $\left.\begin{array}{l} x = 7 \\ y = \dfrac{2x - 8}{3} = 2 \\ z = 3x + y - 12 = 21 + 2 - 12 = 11 \end{array}\right\}$ $\begin{array}{l} x = 7 \\ y = 2 \\ z = 11 \end{array}$

b) $\left.\begin{array}{l} 3x + 4y = 0 \\ 2y = -6 \\ 5x + y - z = 17 \end{array}\right\}$ $\left.\begin{array}{l} y = \dfrac{-6}{2} = -3 \\ x = \dfrac{-4y}{3} = 4 \\ z = 5x + y - 17 = 20 - 3 - 17 = 0 \end{array}\right\}$ $\begin{array}{l} x = 4 \\ y = -3 \\ z = 0 \end{array}$

c) $\left.\begin{array}{l} 3x = -3 \\ 5y = 20 \\ 2x + y - z = -2 \end{array}\right\}$ $\left.\begin{array}{l} x = -1 \\ y = 4 \\ z = 2x + y + 2 = -2 + 4 + 2 = 4 \end{array}\right\}$ $\begin{array}{l} x = -1 \\ y = 4 \\ z = 4 \end{array}$

d) $\left.\begin{array}{r} y = 4 \\ x - z = 11 \\ y - z = 7 \end{array}\right\} \quad \left.\begin{array}{l} y = 4 \\ z = y - 7 = 4 - 7 = -3 \\ x = 11 + z = 11 - 3 = 8 \end{array}\right\} \quad \begin{array}{l} x = 8 \\ y = 4 \\ z = -3 \end{array}$

2. **Resuelve los siguientes sistemas escalonados:**

a) $\left\{\begin{array}{r} y = -5 \\ 2z = 8 \\ 3x = 3 \end{array}\right.$

b) $\left\{\begin{array}{r} x + 2y - z = -3 \\ 3x + y = -5 \\ 5y = -10 \end{array}\right.$

c) $\left\{\begin{array}{r} x - 5y + 3z = 8 \\ 3y - z = 5 \\ 4z = 4 \end{array}\right.$

d) $\left\{\begin{array}{r} 4x + y - z = 7 \\ 2y = 8 \\ 3x = 9 \end{array}\right.$

a) $\left.\begin{array}{r} y = -5 \\ 2z = 8 \\ 3x = 3 \end{array}\right\} \quad \left.\begin{array}{l} y = -5 \\ z = 4 \\ x = 1 \end{array}\right\} \quad \begin{array}{l} x = 1 \\ y = -5 \\ z = 4 \end{array}$

b) $\left.\begin{array}{r} x + 2y - z = -3 \\ 3x + y = -5 \\ 5y = -10 \end{array}\right\} \quad \left.\begin{array}{l} y = \dfrac{-10}{5} = -2 \\ x = \dfrac{-5 - y}{3} = -1 \\ z = x + 2y + 3 = -2 \end{array}\right\} \quad \begin{array}{l} x = -1 \\ y = -2 \\ z = -2 \end{array}$

c) $\left.\begin{array}{r} x - 5y + 3z = 8 \\ 3y - z = 5 \\ 4z = 4 \end{array}\right\} \quad \left.\begin{array}{l} z = 1 \\ y = \dfrac{5 + z}{3} = 2 \\ x = 8 + 5y - 3z = 8 + 10 - 3 = 15 \end{array}\right\} \quad \begin{array}{l} x = 15 \\ y = 2 \\ z = 1 \end{array}$

d) $\left.\begin{array}{r} 4x + y - z = 7 \\ 2y = 8 \\ 3x = 9 \end{array}\right\} \quad \left.\begin{array}{l} x = \dfrac{9}{3} = 3 \\ y = \dfrac{8}{2} = 4 \\ z = 4x + y - 7 = 9 \end{array}\right\} \quad \begin{array}{l} x = 3 \\ y = 4 \\ z = 9 \end{array}$

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3. Resuelve por el método de Gauss:

a) $\begin{cases} x + y + z = 2 \\ x - y + z = 6 \\ x - y - z = 0 \end{cases}$

b) $\begin{cases} 2x + 3y = 14 \\ x - 2y + z = -3 \\ 2x - y - z = 9 \end{cases}$

a) $$\left.\begin{array}{l} x + y + z = 2 \\ x - y + z = 6 \\ x - y - z = 0 \end{array}\right\} \xrightarrow{\begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a + 1.^a \end{array}} \left.\begin{array}{l} x + y + z = 2 \\ 2x + 2z = 8 \\ 2x = 2 \end{array}\right\} \quad \left.\begin{array}{l} x + y + z = 2 \\ x + z = 4 \\ x = 1 \end{array}\right\}$$

$$\left.\begin{array}{l} x = 1 \\ z = 4 - x = 3 \\ y = 2 - x - z = 2 - 1 - 3 = -2 \end{array}\right\} \begin{array}{l} x = 1 \\ y = -2 \\ z = 3 \end{array}$$

b) $$\left.\begin{array}{l} 2x + 3y = 14 \\ x - 2y + z = -3 \\ 2x - y - z = 9 \end{array}\right\} \xrightarrow{\begin{array}{l} 1.^a \\ 2.^a \\ 3.^a + 2.^a \end{array}} \left.\begin{array}{l} 2x + 3y = 14 \\ x - 2y + z = -3 \\ 3x - 3y = 6 \end{array}\right\} \xrightarrow{\begin{array}{l} 1.^a \\ 2.^a \\ 3.^a + 1.^a \end{array}} \left.\begin{array}{l} 2x + 3y = 14 \\ x - 2y + z = -3 \\ 5x = 20 \end{array}\right\}$$

$$\left.\begin{array}{l} x = \dfrac{20}{5} = 4 \\ y = \dfrac{14 - 2x}{3} = 2 \\ z = -3 - x + 2y = -3 - 4 + 4 = -3 \end{array}\right\} \begin{array}{l} x = 4 \\ y = 2 \\ z = -3 \end{array}$$

4. Resuelve:

a) $\begin{cases} 5x - 4y + 3z = 9 \\ 2x + y - 2z = 1 \\ 4x + 3y + 4z = 1 \end{cases}$

b) $\begin{cases} 2x - 5y + 4z = -1 \\ 4x - 5y + 4z = 3 \\ 5x - 3z = 13 \end{cases}$

a) $$\left.\begin{array}{l} 5x - 4y + 3z = 9 \\ 2x + y - 2z = 1 \\ 4x + 3y + 4z = 1 \end{array}\right\} \xrightarrow{\begin{array}{l} 1.^a + 4 \cdot 2.^a \\ 2.^a \\ 3.^a - 3 \cdot 2.^a \end{array}} \left.\begin{array}{l} 13x - 5z = 13 \\ 2x + y - 2z = 1 \\ -2x + 10z = -2 \end{array}\right\} \xrightarrow{\begin{array}{l} 2 \cdot 1.^a + 3.^a \\ 2.^a \\ 3.^a : 2 \end{array}}$$

$$\left.\begin{array}{l} 24x = 24 \\ 2x + y - 2z = 1 \\ -x + 5z = -1 \end{array}\right\} \quad \left.\begin{array}{l} x = 1 \\ z = \dfrac{-1 + x}{5} = 0 \\ y = 1 - 2x + 2z = -1 \end{array}\right\} \begin{array}{l} x = 1 \\ y = -1 \\ z = 0 \end{array}$$

b) $\left.\begin{array}{l} 2x - 5y + 4z = -1 \\ 4x - 5y + 4z = 3 \\ 5x - 3z = 13 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a - 1.^a \\ 3.^a \end{array}$ → $\left.\begin{array}{l} 2x - 5y + 4z = -1 \\ 2x = 4 \\ 5x - 3z = 13 \end{array}\right\}$

$$\left.\begin{array}{l} x = 2 \\ z = \dfrac{5x - 13}{3} = -1 \\ y = \dfrac{2x + 4z + 1}{5} = \dfrac{1}{5} \end{array}\right\} \quad \begin{array}{l} x = 2 \\ y = \dfrac{1}{5} \\ z = -1 \end{array}$$

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5. Intenta resolver por el método de Gauss:

a) $\begin{cases} x + y + z = -2 \\ x - 2y - z = 3 \\ 2x - y = 0 \end{cases}$ **b)** $\begin{cases} x + y + z = -2 \\ x - 2y - z = 3 \\ 2x - y = 1 \end{cases}$

c) $\begin{cases} x + z = 3 \\ 2x - y + 4z = 8 \\ x + y - z = 2 \end{cases}$ **d)** $\begin{cases} x + z = 3 \\ 2x - y + 4z = 8 \\ x + y - z = 1 \end{cases}$

a) $\begin{cases} x + y + z = -2 \\ x - 2y - z = 3 \\ 2x - y = 0 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a \end{array}$ → $\begin{cases} x + y + z = -2 \\ 2x - y = 1 \\ 2x - y = 0 \end{cases}$

Las ecuaciones 2.ª y 3.ª dicen cosas contradictorias (si $2x - y$ es igual a 1, no puede ser igual a 2). Por tanto, el sistema es incompatible.

b) $\begin{cases} x + y + z = -2 \\ x - 2y - z = 3 \\ 2x - y = 1 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a \end{array}$ → $\begin{cases} x + y + z = -2 \\ 2x - y = 1 \\ 2x - y = 1 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a \\ 3.^a - 2.^a \end{array}$ → $\begin{cases} x + y + z = -2 \\ 2x - y = 1 \\ 0 = 0 \end{cases}$

Solo quedan dos ecuaciones. Resolvemos el sistema obteniendo y, z en función de x:

(2.ª) → $y = 2x - 1$

(1.ª) → $z = -2 - y - x = -2 - (2x - 1) - x = -2 - 2x + 1 - x = -3x - 1$

Soluciones: $\begin{cases} y = 2x - 1 \\ z = -3x - 1 \end{cases}$

Para cada valor de x, se obtiene una solución del sistema. Por ejemplo:

Para $x = 0$ → $\begin{cases} x = 0 \\ y = -1 \\ z = -1 \end{cases}$ Para $x = -2$ → $\begin{cases} x = -2 \\ y = -5 \\ z = 5 \end{cases}$

c) $\begin{cases} x \quad\quad + \ \ z = 3 \\ 2x - y + 4z = 8 \\ x + y - \ \ z = 2 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a + 3.^a \\ 3.^a \end{array}$ $\begin{cases} x \quad\quad + \ \ z = 3 \\ 3x \quad\quad + 3z = 10 \\ x + y - \ \ z = 2 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a - 3 \cdot 1.^a \\ 3.^a \end{array}$

$\begin{cases} x \quad\quad + \ \ z = 3 \\ 0x \quad\quad + 0z = 1 \\ x + y - \ \ z = 2 \end{cases}$ La segunda ecuación es absurda. No puede ser $0 = 1$.
Por tanto, el sistema no tiene solución.

d) $\begin{cases} x \quad\quad + \ \ z = 3 \\ 2x - y + 4z = 8 \\ x + y - \ \ z = 1 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a + 3.^a \\ 3.^a \end{array}$ $\begin{cases} x + \quad\quad z = 3 \\ 3x + \quad\quad 3z = 9 \\ x + y - \ \ z = 1 \end{cases}$ $\begin{array}{l} 1.^a \\ 2.^a - 3 \cdot 1.^a \\ 3.^a \end{array}$

$\begin{cases} x + \quad\quad z = 3 \\ 0x + \quad\quad 0z = 0 \\ x + y - \ \ z = 1 \end{cases}$ La segunda ecuación no dice nada. No es una ecuación. Por tanto, solo quedan dos ecuaciones, la 1.ª y la 3.ª.

Resolvemos el sistema resultante dando los valores de x e y en función de z:

$$\begin{cases} x + \quad z = 3 \ \rightarrow \ x = 3 - z \\ x + y - z = 1 \ \rightarrow \ y = 1 - x + z = 1 - (3 - z) + z = -2 + 2z \end{cases}$$

Soluciones: $\begin{cases} x = 3 - z \\ y = -2 + 2z \end{cases}$

Para cada valor que le demos a z, se obtiene una solución del sistema. Por ejemplo:

Para $z = 0 \ \rightarrow \ x = 3, \ y = -2$

Para $z = 4 \ \rightarrow \ x = -1, \ y = 6$

20 **Resuelve:**

a) $\begin{cases} x \cdot y = 15 \\ \dfrac{x}{y} = \dfrac{5}{3} \end{cases}$ b) $\begin{cases} \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{5}{6} \\ 2x + 3y = 2 \end{cases}$

c) $\begin{cases} x^2 + y^2 - 5x - 5y + 10 = 0 \\ x^2 - y^2 - 5x + 5y + 2 = 0 \end{cases}$

☛ ***Suma las dos ecuaciones.***

a) $x = \dfrac{5y}{3}$

$\dfrac{5y^2}{3} = 15;\ \ y^2 = 9 \begin{cases} y = 3 \ \rightarrow\ x = 5 \\ y = -3 \ \rightarrow\ x = -5 \end{cases}$

$x_1 = 5,\ \ y_1 = 3;\ \ x_2 = -5,\ \ y_2 = -3$

b) $\left.\begin{array}{l} 6y + 6x = 5xy \\ y = \dfrac{2-2x}{3} \end{array}\right\}$ $4 - 4x + 6x = \dfrac{5x(2-2x)}{3}$

$6x + 12 = 10x - 10x^2$

$10x^2 - 4x + 12 = 0$

$5x^2 - 2x + 6 = 0$

No tiene solución.

c) $2x^2 - 10x + 12 = 0;\ \ x^2 - 5x + 6 = 0$

$x = \dfrac{5 \pm \sqrt{25-24}}{2} = \dfrac{5 \pm 1}{2} = \begin{cases} 3 \\ 2 \end{cases}$

$$\begin{array}{r} x^2 + y^2 - 5x - 5y + 10 = 0 \\ -x^2 + y^2 + 5x - 5y - \ \ 2 = 0 \\ \hline 2y^2 - \qquad 10y + \ \ 8 = 0 \end{array}$$

$y^2 - 5y + 4 = 0$

$y = \dfrac{5 \pm \sqrt{25-16}}{2} = \dfrac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$x_1 = 3,\ \ y_1 = 4;\ \ x_2 = 3,\ \ y_2 = 1;\ \ x_3 = 2,\ \ y_3 = 4;\ \ x_4 = 2,\ \ y_4 = 1$

21 **Resuelve:**

a) $\begin{cases} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{cases}$

b) $\begin{cases} 2\sqrt{x+1} = y + 1 \\ 2x - 3y = 1 \end{cases}$

c) $\begin{cases} \sqrt{3(x+y)} + x = 12 \\ 2x - y = 6 \end{cases}$

d) $\begin{cases} \sqrt{x+y} + 2 = x + 1 \\ 2x - y = 5 \end{cases}$

e) $\left.\begin{array}{l} x = 25 + y \\ \log \dfrac{y}{x} = -1 \end{array}\right\}$ $\begin{array}{l} y = 0{,}1x \\ 0{,}9x = 25 \end{array}$

$x = \dfrac{250}{9};\ \ y = \dfrac{25}{9}$

f) $\left.\begin{array}{l} \ln x - \ln y = 2 \\ \ln x + \ln y = 4 \end{array}\right\}$ Sumando las dos ecuaciones, queda:
$2 \ln x = 6 \ \to\ \ln x = 3 \ \to\ x = e^3$

Restando a la 2.ª ecuación la 1.ª, queda:

$2 \ln y = 2 \ \to\ \ln y = 1 \ \to\ y = e$

Solución: $x = e^3;\ \ y = e$

Método de Gauss

24 **Resuelve por el método de Gauss:**

a) $\left\{\begin{array}{l} x - y - z = -10 \\ x + 2y + z = 11 \\ 2x - y + z = 8 \end{array}\right.$

b) $\left\{\begin{array}{l} x + y + z = 3 \\ 2x - y + z = 2 \\ x - y + z = 1 \end{array}\right.$

a) $\left.\begin{array}{rl} x - y - z &= -10 \\ x + 2y + z &= 11 \\ 2x - y + z &= 8 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a + 1.^a \end{array}$ $\left.\begin{array}{rl} x - y - z &= -10 \\ 2x + y &= 1 \\ 3x - 2y &= -2 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a \\ 3.^a + 2 \cdot 2.^a \end{array}$

$\left.\begin{array}{rl} x - y - z &= -10 \\ 2x + y &= 1 \\ 7x &= 0 \end{array}\right\}$ $\left.\begin{array}{l} x = 0 \\ y = 1 \\ z = -1 + 10 = 9 \end{array}\right\}$ $\begin{array}{l} x = 0 \\ y = 1 \\ z = 9 \end{array}$

b) $\left.\begin{array}{rl} x + y + z &= 3 \\ 2x - y + z &= 2 \\ x - y + z &= 1 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a + 1.^a \end{array}$ $\left.\begin{array}{rl} x + y + z &= 3 \\ 3x + 2z &= 5 \\ 2x + 2z &= 4 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a \\ 3.^a - 2.^a \end{array}$

$\left.\begin{array}{rl} x + y + z &= 3 \\ 3x + 2z &= 5 \\ -x &= -1 \end{array}\right\}$ $\left.\begin{array}{l} x = 1 \\ z = \dfrac{5 - 3x}{2} = 1 \\ y = 3 - x - z = 1 \end{array}\right\}$ $\begin{array}{l} x = 1 \\ y = 1 \\ z = 1 \end{array}$

25 **Resuelve aplicando el método de Gauss:**

a) $\left\{\begin{array}{l} x+y+z=18\\ x-z=6\\ x-2y+z=0\end{array}\right.$ **b)** $\left\{\begin{array}{l} x+y+z=2\\ 2x+3y+5z=11\\ x-5y+6z=29\end{array}\right.$

a) $$\left.\begin{array}{l} x+y+z=18\\ x-z=6\\ x-2y+z=0\end{array}\right\}\ \begin{array}{l}1.^a\\ 2.^a\\ 3.^a+2\cdot 1.^a\end{array} \longrightarrow \left.\begin{array}{l} x+y+z=18\\ x-z=6\\ 3x+3z=36\end{array}\right\}\ \begin{array}{l}1.^a\\ 2.^a\\ 3.^a:3\end{array} \longrightarrow \left.\begin{array}{l} x+y+z=18\\ x-z=6\\ x+z=12\end{array}\right\}$$

$$\begin{array}{l}1.^a\\ 2.^a\\ 3.^a+2.^a\end{array} \longrightarrow \left.\begin{array}{l} x+y+z=18\\ x-z=6\\ 2x=18\end{array}\right\}\ \left.\begin{array}{l} x=9\\ z=x-6=3\\ y=18-x-z=6\end{array}\right\}\ \begin{array}{l} x=9\\ y=6\\ z=3\end{array}$$

b) $$\left.\begin{array}{l} x+y+z=2\\ 2x+3y+5z=11\\ x-5y+6z=29\end{array}\right\}\ \begin{array}{l}1.^a\\ 2.^a-2\cdot 1.^a\\ 3.^a-1.^a\end{array} \longrightarrow \left.\begin{array}{l} x+y+z=2\\ y+3z=7\\ -6y+5z=27\end{array}\right\}\ \begin{array}{l}1.^a\\ 2.^a\\ 3.^a+6\cdot 2.^a\end{array} \longrightarrow$$

$$\left.\begin{array}{l} x+y+z=2\\ y+3z=7\\ 23z=69\end{array}\right\}\ \left.\begin{array}{l} z=\dfrac{69}{23}=3\\ y=7-3z=7-9=-2\\ x=2-y-z=2+2-3=1\end{array}\right\}\ \begin{array}{l} x=1\\ y=-2\\ z=3\end{array}$$

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26 **Resuelve por el método de Gauss:**

a) $\left\{\begin{array}{l} x+y-2z=9\\ 2x-y+4z=4\\ 2x-y+6z=-1\end{array}\right.$

b) $\left\{\begin{array}{l} 2x-3y+z=0\\ 3x+6y-2z=0\\ 4x+y-z=0\end{array}\right.$

a) $$\left.\begin{array}{l} x+y-2z=9\\ 2x-y+4z=4\\ 2x-y+6z=-1\end{array}\right\}\ \begin{array}{l}1.^a\\ 2.^a+1.^a\\ 3.^a+1.^a\end{array} \longrightarrow \left.\begin{array}{l} x+y-2z=9\\ 3x+2z=13\\ 3x+4z=8\end{array}\right\}\ \begin{array}{l}1.^a\\ 2.^a\\ 3.^a-2.^a\end{array} \longrightarrow$$

$$\left.\begin{array}{l} x+y-2z=9\\ 3x+2z=13\\ 2z=-5\end{array}\right\}\ \left.\begin{array}{l} z=\dfrac{-5}{2}\\ x=\dfrac{13-2z}{3}=6\\ y=9-x+2z=9-6-5=-2\end{array}\right\}\ \begin{array}{l} x=6\\ y=-2\\ z=\dfrac{-5}{2}\end{array}$$

b) $\left.\begin{array}{l} 2x - 3y + z = 0 \\ 3x + 6y - 2z = 0 \\ 4x + y - z = 0 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a + 2 \cdot 1.^a \\ 3.^a + 1.^a \end{array}$ $\left.\begin{array}{l} 2x - 3y + z = 0 \\ 7x = 0 \\ 6x - 2y = 0 \end{array}\right\}$ $\begin{array}{l} x = 0 \\ y = 0 \\ z = 0 \end{array}$

27 **Resuelve aplicando el método de Gauss:**

a) $\left\{\begin{array}{l} x - y = 1 \\ 2x + 6y - 5z = -4 \\ x + y - z = 0 \end{array}\right.$ **b)** $\left\{\begin{array}{l} x + 2y + z = 3 \\ x - 2y + 5z = 5 \\ 5x - 2y + 17z = 1 \end{array}\right.$

c) $\left\{\begin{array}{l} x + y + 3z = 2 \\ 2x + 3y + 4z = 1 \\ -2x - y - 8z = -7 \end{array}\right.$ **d)** $\left\{\begin{array}{l} 2x - y - z = 2 \\ 3x - 2y - 2z = 2 \\ -5x + 3y + 5z = -1 \end{array}\right.$

e) $\left\{\begin{array}{l} x + y + z = 3 \\ -x + 2y + z = 5 \\ x + 4y + 3z = 1 \end{array}\right.$ **f)** $\left\{\begin{array}{l} -2x + y + z = 1 \\ 3x + 2y - z = 0 \\ -x + 4y + z = 2 \end{array}\right.$

☛ ***Encontrarás sistemas compatibles (determinados e indeterminados) y sistemas incompatibles.***

a) $\left.\begin{array}{l} x - y = 1 \\ 2x + 6y - 5z = -4 \\ x + y - z = 0 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a - 5 \cdot 3.^a \\ 3.^a \end{array}$ $\left.\begin{array}{l} x - y = 1 \\ -3x + y = -4 \\ x + y - z = 0 \end{array}\right\}$

$\begin{array}{l} 1.^a \\ 2.^a + 3 \cdot 1.^a \\ 3.^a \end{array}$ $\left.\begin{array}{l} x - y = 1 \\ -2y = -1 \\ x + y - z = 0 \end{array}\right\}$ $\begin{array}{l} y = \dfrac{1}{2} \\ x = 1 + \dfrac{1}{2} = \dfrac{3}{2} \\ z = \dfrac{3}{2} + \dfrac{1}{2} = 2 \end{array}$ $\left.\begin{array}{l} x = \dfrac{3}{2} \\ y = \dfrac{1}{2} \\ z = 2 \end{array}\right.$

b) $\left.\begin{array}{l} x + 2y + z = 3 \\ x - 2y + 5z = 5 \\ 5x - 2y + 17z = 1 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a + 1.^a \end{array}$ $\left.\begin{array}{l} x + 2y + z = 3 \\ 2x + 6z = 8 \\ 6x + 18z = 4 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a : 2 \\ 3.^a : 6 \end{array}$

$\left.\begin{array}{l} x + 2y + z = 3 \\ x + 3z = 4 \\ x + 3z = 4/6 \end{array}\right\}$ Las ecuaciones 2.ª y 3.ª dicen cosas contradictorias.
El sistema es incompatible, no tiene solución.

c) $\left.\begin{array}{l} x + y + 3z = 2 \\ 2x + 3y + 4z = 1 \\ -2x - y - 8z = -7 \end{array}\right\}$ $\begin{array}{l} 1.^a \\ 2.^a - 3 \cdot 1.^a \\ 3.^a + 1.^a \end{array}$ $\left.\begin{array}{l} x + y + 3z = 2 \\ -x - 5z = -5 \\ -x - 5z = -5 \end{array}\right\}$

Hay dos ecuaciones iguales. El sistema es compatible indeterminado. Buscamos las soluciones en función de z:

$$\left.\begin{array}{l} x + y = 2 - 3z \\ -x = -5 + 5z \end{array}\right\} \begin{array}{l} \to (5 - 5z) + y = 2 - 3z \to y = 2z - 3 \\ \to x = 5 - 5z \end{array}$$

Solución: $x = 5 - 5z, \ y = 2z - 3, \ z = z$

d) $$\left.\begin{array}{rl} 2x - y - z &= 2 \\ 3x - 2y - 2z &= 2 \\ -5x + 3y + 5z &= -1 \end{array}\right\} \begin{array}{l} 1.^a \\ 2.^a - 2 \cdot 1.^a \\ 3.^a + 5 \cdot 1.^a \end{array} \Rightarrow \left.\begin{array}{rl} 2x - y - z &= 2 \\ -x &= -2 \\ 5x - 2y &= 9 \end{array}\right\} \left.\begin{array}{l} x = 2 \\ y = \dfrac{5x - 9}{2} = \dfrac{1}{2} \\ z = 2x - y - 2 = \dfrac{3}{2} \end{array}\right\}$$

Solución: $x = 2, \ y = \dfrac{1}{2}, \ z = \dfrac{3}{2}$

e) $$\left.\begin{array}{rl} x + y + z &= 3 \\ -x + 2y + z &= 5 \\ x + 4y + 3z &= 1 \end{array}\right\} \begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a - 1.^a \end{array} \Rightarrow \left.\begin{array}{rl} x + y + z &= 3 \\ 3y + 2z &= 8 \\ 3y + 2z &= -2 \end{array}\right\}$$

Las ecuaciones 2.ª y 3.ª obtenidas dicen cosas contradictorias. Por tanto, el sistema es incompatible.

f) $$\left.\begin{array}{rl} -2x + y + z &= 1 \\ 3x + 2y - z &= 0 \\ -x + 4y + z &= 2 \end{array}\right\} \begin{array}{l} 1.^a \\ 2.^a + 1.^a \\ 3.^a - 1.^a \end{array} \Rightarrow \left.\begin{array}{rl} -2x + y + z &= 1 \\ x + 3y &= 1 \\ x + 3y &= 1 \end{array}\right\}$$

Hay dos ecuaciones iguales. El sistema es compatible indeterminado. Buscamos las soluciones en función del parámetro y:

$$\left.\begin{array}{l} -2x + z = 1 - y \\ x = 1 - 3y \end{array}\right\} \to -2(1 - 3y) + z = 1 - y \to z = 3 - 7y$$

Solución: $x = 1 - 3y, \ z = 3 - 7y$

46 **Resuelve:**

a) $\begin{cases} \sqrt{x+y} - \sqrt{x-y} = \sqrt{2y} \\ x + y = 8 \end{cases}$

b) $\begin{cases} \sqrt{4y+2x} = \sqrt{3y+x} - 1 \\ y + x = -5 \end{cases}$

c) $\begin{cases} (x+3)\,(y-5) = 0 \\ (x-2)\,(y-1) = 0 \end{cases}$

a) $x = 8 - y$

$\sqrt{8} - \sqrt{8-2y} = \sqrt{2y} \ \rightarrow\ \sqrt{8} - \sqrt{2y} = \sqrt{8-2y} \ \rightarrow$

$\rightarrow\ 8 + 2y - 2\sqrt{16y} = 8 - 2y \ \rightarrow\ 2y - 8\sqrt{y} = -2y \ \rightarrow$

$\rightarrow\ 4y = 8\sqrt{y} \ \rightarrow\ 16y^2 = 64y \ \rightarrow\ 16y^2 - 64y = 0 \ \rightarrow$

$\rightarrow\ 16y(y-4) = 0 \ \begin{cases} y = 0 \ \rightarrow\ x = 8 \\ y = 4 \ \rightarrow\ x = 4 \end{cases}$

$x_1 = 8,\ \ y_1 = 0;\ \ x_2 = 4,\ \ y_2 = 4$

b) $x = -5 - y$

$\sqrt{4y - 10 - 2y} = \sqrt{3y - 5 - y} - 1$

$\sqrt{2y - 10} = \sqrt{2y - 5} - 1$

$2y - 10 = 2y - 5 + 1 - 2\sqrt{2y - 5}$

$2\sqrt{2y - 5} = 6$

$\sqrt{2y - 5} = 3$

$2y - 5 = 9$

$x = -12;\ \ y = 7$

c) $x_1 = -3,\ \ y_1 = 1;\ \ x_2 = 2,\ \ y_2 = 5$