SISTEMAS DE ECUACIONES

EJERCICIO 6 : Halla la solución de los siguientes sistemas, analítica y gráficamente:

a) $\left.\begin{array}{l}\frac{x}{3}+\frac{y}{2}=3\\ \frac{x}{2}+\frac{y}{2}=4\end{array}\right\}$ **b)** $\left.\begin{array}{l}y-4x-2=0\\ y=x^2+3x\end{array}\right\}$ **c)** $\left.\begin{array}{l}y=x^2-2x\\ y+x-6=0\end{array}\right\}$ **d)** $\left.\begin{array}{l}\frac{x-1}{3}+\frac{y}{2}=2\\ 3x+y=7\end{array}\right\}$ **e)** $\left.\begin{array}{l}y=x^2-3x\\ y-2x+6=0\end{array}\right\}$

Solución:

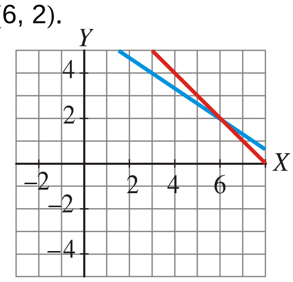
a)

• Resolvemos el sistema analíticamente: $\left.\begin{array}{l}\frac{x}{3}+\frac{y}{2}=3\\ \frac{x}{2}+\frac{y}{2}=4\end{array}\right\}$ $\left.\begin{array}{l}\frac{2x}{6}+\frac{3y}{6}=\frac{18}{6}\\ \frac{x}{2}+\frac{y}{2}=\frac{8}{2}\end{array}\right\}$ $\left.\begin{array}{l}2x+3y=18\\ x+y=8\end{array}\right\}$ $y=8-x$

$2x+3(8-x)=18$; $2x+24-3x=18$; $-x=-6$; $x=6$ $\rightarrow$ $y=8-6=2$; *Solución:* $x=6$; $y=2$

• Interpretación gráfica: $\left.\begin{array}{l}\frac{x}{3}+\frac{y}{2}=3 \rightarrow y=\frac{18-2x}{3}=6-\frac{2}{3}x=\frac{-2}{3}x+6\\ \frac{x}{2}+\frac{y}{2}=4 \rightarrow y=8-x\end{array}\right\}$

Estas dos rectas se cortan en el punto (6, 2).

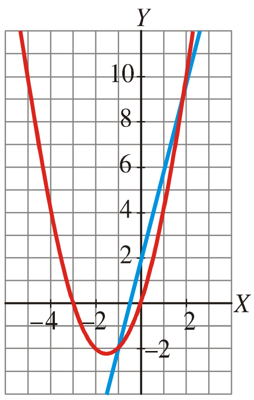


b)

• Lo resolvemos analíticamente: $\left.\begin{array}{l}y-4x-2=0\\ y=x^2+3x\end{array}\right\}$ $\begin{array}{l}y=4x+2\\ 4x+2=x^2+3x;\quad 0=x^2-x-2\end{array}$

$x=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm\sqrt{9}}{2}=\frac{1\pm 3}{2}\rightarrow\left\{\begin{array}{l}x=2 \rightarrow y=10\\ x=-1 \rightarrow y=-2\end{array}\right.$ *Solución* : $\left.\begin{array}{l}x_1=2\\ y_1=10\end{array}\right\}$ y $\left.\begin{array}{l}x_2=-1\\ y_2=-2\end{array}\right\}$

• Interpretación gráfica: $\left.\begin{array}{l}y=4x+2\\ y=x^2+3x\end{array}\right\}$ La recta y la parábola se cortan en los puntos $(2,10)$ y $(-1,-2)$.

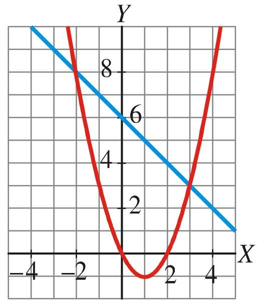


c)

• Resolvemos analíticamente el sistema: $\left.\begin{array}{l} y = x^2 - 2x \\ y + x - 6 = 0 \end{array}\right\}$ $\begin{array}{l} y = x^2 - 2x \\ x^2 - 2x + x - 6 = 0; \quad x^2 - x - 6 = 0 \end{array}$

$$x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm \sqrt{25}}{2} = \frac{1 \pm 5}{2} \rightarrow \left\{\begin{array}{l} x = 3 \rightarrow y = 3 \\ x = -2 \rightarrow y = 8 \end{array}\right. \qquad \textit{Solución}: \left.\begin{array}{l} x_1 = 3 \\ y_1 = 3 \end{array}\right\} \text{ y } \left.\begin{array}{l} x_2 = -2 \\ y_2 = 8 \end{array}\right\}$$

• Interpretación gráfica: $\left.\begin{array}{l} y = x^2 - 2x \\ y = 6 - x \end{array}\right\}$ La parábola y la recta se cortan en los puntos $(3, 3)$ y $(-2, 8)$.



d)

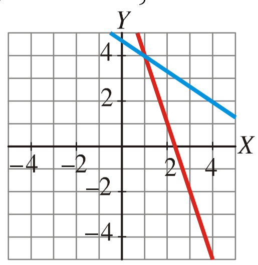
• Resolvemos analíticamente el sistema: $\left.\begin{array}{l} \frac{x-1}{3} + \frac{y}{2} = 2 \\ 3x + y = 7 \end{array}\right\}$ $\left.\begin{array}{l} \frac{2x-2}{6} + \frac{3y}{6} = \frac{12}{6} \\ 3x + y = 7 \end{array}\right\}$ $\left.\begin{array}{l} 2x - 2 + 3y = 12 \\ 3x + y = 7 \end{array}\right\}$

$\left.\begin{array}{l} 2x + 3y = 14 \\ 3x + y = 7 \end{array}\right\}$ $y = 7 - 3x; \quad 2x + 3(7 - 3x) = 14$

$2x + 21 - 9x = 14; \quad 2x - 9x = 14 - 21; \quad -7x = -7; \quad x = 1; \quad y = 7 - 3 \cdot 1 = 7 - 3 = 4$

Solución: $x = 1; \quad y = 4$

• Interpretación gráfica: $\left.\begin{array}{l} 2x + 3y = 14 \rightarrow y = \frac{14 - 2x}{3} \\ 3x + y = 7 \rightarrow y = 7 - 3x \end{array}\right\}$ Estas dos rectas se cortan en el punto (1, 4).

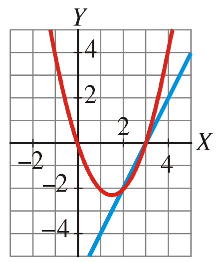


e)

• Lo resolvemos analíticamente: $\left.\begin{array}{l} y = x^2 - 3x \\ y - 2x + 6 = 0 \end{array}\right\}$ $\begin{array}{l} y = x^2 - 3x \\ x^2 - 3x - 2x + 6 = 0; \quad x^2 - 5x + 6 = 0 \end{array}$

$$x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2} \rightarrow \left\{\begin{array}{l} x = 3 \rightarrow y = 0 \\ x = 2 \rightarrow y = -2 \end{array}\right. \qquad \textit{Solución}: \left.\begin{array}{l} x_1 = 3 \\ y_1 = 0 \end{array}\right\} \text{ y } \left.\begin{array}{l} x_2 = 2 \\ y_2 = -2 \end{array}\right\}$$

• Interpretación gráfica: $\left.\begin{array}{l} y = x^2 - 3x \\ y = 2x - 6 \end{array}\right\}$ La parábola y la recta se cortan en los puntos $(3, 0)$ y $(2, -2)$



EJERCICIO 7 : Halla las soluciones de estos sistemas:

a) $\left.\begin{array}{l} y=3x+1 \\ \sqrt{x+y+4}=y-x \end{array}\right\}$ b) $\left.\begin{array}{l} \frac{3}{x}-\frac{x}{y}=0 \\ 2x-y=3 \end{array}\right\}$ c) $\left.\begin{array}{l} \frac{2}{x}+\frac{3}{y}=3 \\ x+y=4 \end{array}\right\}$ d) $\left.\begin{array}{l} 2x+y=6 \\ \sqrt{x}-y=-3 \end{array}\right\}$

e) $\left.\begin{array}{l} \frac{1}{x+y}=\frac{2}{5} \\ \frac{1}{x}+\frac{1}{y}=\frac{5}{2} \end{array}\right\}$ f) $\left.\begin{array}{l} 2log\ x+log\ y=1 \\ log\ x-2log\ y=-2 \end{array}\right\}$ g) $\left.\begin{array}{l} 2^{x+y}=32 \\ ln\ x+ln\ y=ln\ 6 \end{array}\right\}$ h) $\left.\begin{array}{l} 2log\ x-log\ y=0 \\ 2^{y+2x}=8 \end{array}\right\}$

i) $\left.\begin{array}{l} y^2-x=2 \\ log\ (x+y)=1 \end{array}\right\}$ j) $\left.\begin{array}{l} 2^{x+1}+2^y=8 \\ log\ y-log\ x=log\ 2 \end{array}\right\}$ k) $\left.\begin{array}{l} x-y=9 \\ log\,x-log\,y=1 \end{array}\right\}$ l) $\left.\begin{array}{l} y^2-x^2=-3 \\ xy=-2 \end{array}\right\}$

m) $\left.\begin{array}{l} 3\sqrt{x+1}=y-2 \\ 3x+y=-1 \end{array}\right\}$ n) $\left.\begin{array}{l} \frac{1}{x}-\frac{1}{y}=\frac{1}{6} \\ 2x-y=1 \end{array}\right\}$ ñ) $\left.\begin{array}{l} x-2y=0 \\ 2^x+2^y=6 \end{array}\right\}$ o) $\left.\begin{array}{l} x-2y=-1 \\ \frac{1}{x}+\frac{1}{y}=\frac{5}{6} \end{array}\right\}$

p) $\left.\begin{array}{l} x^2+y^2=13 \\ xy=6 \end{array}\right\}$ q) $\left.\begin{array}{l} y=5-\sqrt{x} \\ x=y^2-2y+1 \end{array}\right\}$

Solución:

a) $\left.\begin{array}{l} y=3x+1 \\ \sqrt{x+y+4}=y-x \end{array}\right\}$ $\begin{array}{l} y=3x+1 \\ \sqrt{x+3x+1+4}=3x+1-x \end{array}$ $\sqrt{4x+5}=2x+1;\quad 4x+5=(2x+1)^2$

$4x+5=4x^2+1+4x;\quad 4=4x^2;\quad x^2=1;\ x=\pm\sqrt{1}\ \rightarrow \left\{\begin{array}{l} x=-1 \ \rightarrow \ \text{no válida} \\ x=1 \ \rightarrow \ y=4 \end{array}\right.$

Hay una solución: $x=1;\quad y=4$

b) $\left.\begin{array}{l} \frac{3}{x}-\frac{x}{y}=0 \\ 2x-y=3 \end{array}\right\}$ $\left.\begin{array}{l} 3y-x^2=0 \\ 2x-y=3 \end{array}\right\}$ $\begin{array}{l} y=\frac{x^2}{3} \\ 2x-\frac{x^2}{3}=3;\quad 6x-x^2=9 \end{array}$

$0=x^2-6x+9;\quad x=\frac{6\pm\sqrt{36-36}}{2}=\frac{6}{2}=3\ \rightarrow\ y=3$ *Solución:* $x=3;\ \ y=3$

c) $\left.\begin{array}{l} \frac{2}{x}+\frac{3}{y}=3 \\ x+y=4 \end{array}\right\}$ $\left.\begin{array}{l} \frac{2}{x}+\frac{3}{4-x}=3 \\ y=4-x \end{array}\right\}$ $\frac{2(4-x)}{x(4-x)}+\frac{3x}{x(4-x)}=\frac{3x(4-x)}{x(4-x)};$

$8-2x+3x=12x-3x^2;\quad 3x^2-11x+8=0$

$x=\frac{11\pm\sqrt{121-96}}{6}=\frac{11\pm\sqrt{25}}{6}=\frac{11\pm5}{6}\ \rightarrow \left\{\begin{array}{l} x=\frac{16}{6}=\frac{8}{3}\ \rightarrow\ y=\frac{4}{3} \\ x=1\ \rightarrow\ y=3 \end{array}\right.$

Hay dos soluciones: $\left.\begin{array}{l} x_1=\frac{8}{3} \\ y_1=\frac{4}{3} \end{array}\right\}$ y $\left.\begin{array}{l} x_2=1 \\ y_2=3 \end{array}\right\}$

d) $\left.\begin{array}{l} 2x+y=6 \\ \sqrt{x}-y=-3 \end{array}\right\}$ $\left.\begin{array}{l} y=6-2x \\ \sqrt{x}+3=y \end{array}\right\}$ $\begin{array}{l} 6-2x=\sqrt{x}+3 \\ 3-2x=\sqrt{x} \end{array}$ $(3-2x)^2=(\sqrt{x})^2$; $9+4x^2-12x=x$; $4x^2-13x+9=0$

$$x=\frac{13\pm\sqrt{169-144}}{8}=\frac{13\pm\sqrt{25}}{8}=\frac{13\pm 5}{8} \rightarrow \begin{cases} x=\frac{18}{8}=\frac{9}{4} \rightarrow \text{no válida} \\ x=1 \rightarrow y=4 \end{cases}$$

$\left(\text{La solución } x=\frac{9}{4} \text{ no es válida, puesto que } 3-2\cdot\frac{9}{4}=-\frac{3}{2}\neq\sqrt{\frac{9}{4}}=\frac{3}{2}\right)$

La única solución del sistema es $x=1$, $y=4$.

e) $\left.\begin{array}{l} \frac{1}{x+y}=\frac{2}{5} \\ \frac{1}{x}+\frac{1}{y}=\frac{5}{2} \end{array}\right\}$ $\left.\begin{array}{l} 5=2(x+y) \\ 2y+2x=5xy \end{array}\right\}$ $\begin{array}{l} 5=2x+2y \\ 5=5xy \rightarrow 1=xy \rightarrow y=\frac{1}{x} \end{array}$

$5=2x+\frac{2}{x}$; $5x=2x^2+2$; $0=2x^2-5x+2$

$$x=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm\sqrt{9}}{4}=\frac{5\pm 3}{4} \rightarrow \begin{cases} x=2 \rightarrow y=\frac{1}{2} \\ x=\frac{2}{4}=\frac{1}{2} \rightarrow y=2 \end{cases}$$

Hay dos soluciones: $\left.\begin{array}{l} x_1=2 \\ y_1=\frac{1}{2} \end{array}\right\}$ y $\left.\begin{array}{l} x_2=\frac{1}{2} \\ y_2=2 \end{array}\right\}$

f) $\left.\begin{array}{l} 2\log x+\log y=1 \\ \log x-2\log y=2 \end{array}\right\}$ $\left.\begin{array}{l} 2(2\log x+\log y)=2 \\ \log x-2\log y=-2 \end{array}\right\}$ $\begin{array}{l} 4\log x+2\log y=2 \\ \log x-2\log y=-2 \\ \hline 5\log x \qquad\quad =0 \end{array}$ $\rightarrow \log x=0 \rightarrow x=1$

Sustituyendo en la primera ecuación este valor, queda: $2\log x+\log y=1 \rightarrow \log y=1 \rightarrow y=10$

Por tanto, la solución es $x=1$, $y=10$.

g) $\left.\begin{array}{l} 2^{x+y}=32 \\ \ln x+\ln y=\ln 6 \end{array}\right\}$ $\left.\begin{array}{l} 2^{x+y}=2^5 \\ \ln(xy)=\ln 6 \end{array}\right\}$ $\left.\begin{array}{l} x+y=5 \\ xy=6 \end{array}\right\}$ $\begin{array}{l} y=5-x \\ x(5-x)=6 \end{array}$

$$5x-x^2=6 \rightarrow 0=x^2-5x+6 \rightarrow x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm\sqrt{1}}{2}=\frac{5\pm 1}{2} \rightarrow \begin{cases} x=3 \rightarrow y=5-3=2 \\ x=2 \rightarrow y=5-2=3 \end{cases}$$

Hay dos soluciones: $x_1=3$, $y_1=2$; $x_2=2$, $y_2=3$

h) $\left.\begin{array}{l} 2\log x-\log y=0 \\ 2^{y+2x}=8 \end{array}\right\}$ $\left.\begin{array}{l} \log x^2=\log y \\ 2^{y+2x}=2^3 \end{array}\right\}$ $\left.\begin{array}{l} x^2=y \\ y+2x=3 \end{array}\right\}$ $\left.\begin{array}{l} x^2=y \\ y=3-2x \end{array}\right\}$ $x^2=3-2x \rightarrow x^2+2x-3=0$

$$x=\frac{-2\pm\sqrt{4+12}}{2}=\frac{-2\pm\sqrt{16}}{2}=\frac{-2\pm 4}{2} \rightarrow \begin{cases} x=1 \rightarrow y=1 \\ x=-3 \text{ (no válida)} \end{cases}$$

Hay una única solución: $x=1$, $y=1$

i) $\left.\begin{array}{l} y^2 - x = 2 \\ \log(x+y) = 1 \end{array}\right\}$ $\quad \begin{array}{l} y^2 - 2 = x \\ \log(y^2 - 2 + y) = 1 \rightarrow \quad y^2 - 2 + y = 10 \end{array}$

$y^2 + y - 12 = 0 \quad \rightarrow \quad y = \dfrac{-1 \pm \sqrt{1+48}}{2} = \dfrac{1 \pm \sqrt{49}}{2} = \dfrac{-1 \pm 7}{2} \quad \rightarrow \quad \begin{cases} y = 3 \\ y = -4 \end{cases}$

• $y = 3 \quad \rightarrow \quad x = 9 - 2 = 7$

• $y = -4 \quad \rightarrow \quad x = 16 - 2 = 14$

Hay dos soluciones: $x_1 = 7,\ y_1 = 3$; $x_2 = 14,\ y_2 = -4$

j) $\left.\begin{array}{l} 2^{x+1} + 2^y = 8 \\ \log y - \log x = \log 2 \end{array}\right\}$ $\left.\begin{array}{l} 2^{x+1} + 2^y = 8 \\ \log \dfrac{y}{x} = \log 2 \end{array}\right\}$ $\left.\begin{array}{l} 2^{x+1} + 2^y = 8 \\ \dfrac{y}{x} = 2 \end{array}\right\}$ $\quad y = 2x$

$2^{x+1} + 2^{2x} = 8 \quad \rightarrow \quad 2^x \cdot 2 + (2^x)^2 = 8$; Cambio: $2^x = z \quad \rightarrow \quad 2z + z^2 = 8 \quad \rightarrow z^2 + 2z - 8 = 0$

$z = \dfrac{-2 \pm \sqrt{4+32}}{2} = \dfrac{-2 \pm \sqrt{36}}{2} = \dfrac{-2 \pm 6}{2} \quad \rightarrow \quad \begin{cases} z = 2 \\ z = -4 \end{cases}$

• $z = 2 \quad \rightarrow \quad 2^x = 2 \quad \rightarrow \quad x = 1 \quad \rightarrow \quad y = 2$

• $z = -4 \quad \rightarrow \quad 2^x = -4 \quad \rightarrow$ No vale

El sistema tiene una única solución: $x = 1,\ y = 2$

k) $\left.\begin{array}{r} x - y = 9 \\ \log x - \log y = 1 \end{array}\right\}$ $\left.\begin{array}{l} x = 9 + y \\ \log \dfrac{x}{y} = 1 \end{array}\right\}$ $\left.\begin{array}{l} x = 9 + y \\ \dfrac{x}{y} = 10 \end{array}\right\}$ $\left.\begin{array}{l} x = 9 + y \\ x = 10y \end{array}\right\}$ $9 + y = 10y \quad \rightarrow \quad 9 = 9y \quad \rightarrow \quad y = 1 \quad \rightarrow \quad x = 10$

Hay una solución: $x = 10;\ y = 1$

l) $\left.\begin{array}{r} y^2 - x^2 = -3 \\ xy = -2 \end{array}\right\}$ $\left.\begin{array}{r} y^2 - x^2 = -3 \\ y = \dfrac{-2}{x} \end{array}\right\}$ $\left(\dfrac{-2}{x}\right)^2 - x^2 = -3$; $\dfrac{4}{x^2} - x^2 = -3 \quad \rightarrow \quad 4 - x^4 = -3x^2 \quad \rightarrow \quad 0 = x^4 - 3x^2 - 4$

Cambio: $x^2 = z \quad \rightarrow \quad z^2 - 3z - 4 = 0$

$z = \dfrac{3 \pm \sqrt{9+16}}{2} = \dfrac{3 \pm \sqrt{25}}{2} = \dfrac{3 \pm 5}{2} \quad \rightarrow \quad \begin{cases} z = 4 \quad \rightarrow \quad x^2 = 4 \quad \rightarrow \quad x = \pm\sqrt{4} = \pm 2 \\ z = -1 \quad \rightarrow \quad \text{no vale} \end{cases}$

• $x = 2 \quad \rightarrow \quad y = -1$ Hay dos soluciones: $x_1 = 2;\ y_1 = -1$

• $x = -2 \quad \rightarrow \quad y = 1$ $\quad x_2 = -2;\ y_2 = 1$

m) $\left.\begin{array}{l} 3\sqrt{x+1} = y - 2 \\ 3x + y = -1 \end{array}\right\}$ $\left.\begin{array}{l} 3\sqrt{x+1} = y - 2 \\ y = -1 - 3x \end{array}\right\}$ $3\sqrt{x+1} = -1 - 3x - 2$

$3\sqrt{x+1} = -3x - 3 \quad \rightarrow \quad \sqrt{x+1} = \dfrac{-3x - 3}{3} \quad \rightarrow \quad \sqrt{x+1} = -x - 1$

$x + 1 = (-x - 1)^2 \quad \rightarrow \quad x + 1 = x^2 + 2x + 1 \quad \rightarrow \quad 0 = x^2 + x \Rightarrow x(x+1) = 0 \quad \rightarrow \quad \begin{cases} x = 0 \quad \rightarrow \quad \text{no válida} \\ x = -1 \quad \rightarrow \quad y = 2 \end{cases}$

Hay una única solución: $x = -1;\ y = 2$

n) $\left.\begin{array}{l} \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{1}{6} \\ 2x - y = 1 \end{array}\right\}$ $\left.\begin{array}{l} 6y - 6x = xy \\ 2x - 1 = y \end{array}\right\}$ $6(2x - 1) - 6x = x(2x - 1) \Rightarrow 12x - 6 - 6x = 2x^2 - x \quad \rightarrow \quad 0 = 2x^2 - 7x + 6$

$x = \dfrac{7 \pm \sqrt{49 - 48}}{4} = \dfrac{7 \pm \sqrt{1}}{4} = \dfrac{7 \pm 1}{4} \quad \rightarrow \quad \begin{cases} x = 2 \quad \rightarrow \quad y = 3 \\ x = \dfrac{6}{4} = \dfrac{3}{2} \quad \rightarrow \quad y = 2 \end{cases}$

Hay dos soluciones: $x_1 = 2;\ y_1 = 3$; $x_2 = \dfrac{3}{2};\ y_2 = 2$

ñ) $\left.\begin{array}{l} x-2y=0 \\ 2^x+2^y=6 \end{array}\right\} \quad \left.\begin{array}{l} x=2y \\ 2^{2y}+2^y=6 \end{array}\right\} \quad (2^y)^2+2^y=6$ Hacemos el cambio: $2^y=z$

$$z^2+z-6=0 \rightarrow z=\frac{-1\pm\sqrt{1+24}}{2}=\frac{-1\pm\sqrt{25}}{2}=\frac{-1\pm5}{2} \rightarrow \begin{cases} z=2 \\ z=-3 \end{cases}$$

• $z=2 \rightarrow 2^y=2 \rightarrow y=1 \rightarrow x=2$

• $z=-3 \rightarrow 2^y=-3 \rightarrow$ no válida

Hay una solución: $x=2;\ y=1$

$$\frac{1}{x}+\frac{1}{y}=\frac{5}{6} \Rightarrow 6(y+x)=5xy$$

o) $x=-1+2y \Rightarrow 6y+6x=5xy \Rightarrow 6y+6(-1+2y)=5(-1+2y)y$

$$6y-6+12y=-5y+10y^2 \Rightarrow 10y^2-23y+6=0$$

$$y=\frac{23\pm\sqrt{529-240}}{20}=\frac{23\pm17}{20}\begin{cases} y=2 \rightarrow x=3 \\ y=\frac{6}{20}=\frac{3}{10} \rightarrow x=\frac{-2}{5} \end{cases}$$

p) $y=\frac{6}{x} \rightarrow x^2+\frac{36}{x^2}=13 \rightarrow x^4+36=13x^2 \rightarrow x^4-13x^2+36=0$

Cambio: $x^2=z.$ Así: $z^2-13z+36=0 \Rightarrow z=\frac{13\pm\sqrt{169-144}}{2}=\frac{13\pm\sqrt{25}}{2}=\frac{13\pm5}{2}\begin{cases} z=9 \rightarrow x=\pm3 \\ z=4 \rightarrow x=\pm2 \end{cases}$

Soluciones: $\begin{cases} x_1=-3 \\ y_1=-2 \end{cases} \begin{cases} x_2=3 \\ y_2=2 \end{cases} \begin{cases} x_3=-2 \\ y_3=-3 \end{cases} \begin{cases} x_4=2 \\ y_4=3 \end{cases}$

q) $x=(5-\sqrt{x})^2-2(5-\sqrt{x})+1 \Rightarrow x=25+x-10\sqrt{x}-10+2\sqrt{x}+1 \Rightarrow$

$8\sqrt{x}=16 \Rightarrow \sqrt{x}=2 \Rightarrow x=4,\quad y=3$

SISTEMAS DE ECUACIONES. MÉTODO DE GAUSS

EJERCICIO 8 : Obtén, mediante el método de Gauss, la solución de los siguientes sistemas de ecuaciones:

a) $\begin{cases} 3x+2y+z=7 \\ 2x-2y-z=8 \\ x+5y+z=-2 \end{cases}$ **b)** $\begin{cases} 3x+y-2z=-6 \\ 2x-y+3z=-8 \\ x+y-z=4 \end{cases}$ **c)** $\begin{cases} -2x-y+z=-4 \\ 3x+y-2z=6 \\ 2x+y+z=6 \end{cases}$

d) $\begin{cases} 2x-y+2z=2 \\ x+2y-z=3 \\ 2x-y+3z=1 \end{cases}$ **e)** $\begin{cases} x+2y-2z=6 \\ x-3y+z=-7 \\ 2x-y+z=-3 \end{cases}$ **f)** $\begin{cases} x+y-z=2 \\ 2x-2y+3z=1 \\ x+2y-z=4 \end{cases}$

g) $\begin{cases} x-2y+z=6 \\ 3x+y-z=7 \\ x-y+2z=6 \end{cases}$ **h)** $\begin{cases} x-y+2z=7 \\ x+y-3z=-5 \\ 2x-y+2z=9 \end{cases}$ **i)** $\begin{cases} x+y+2z=6 \\ x-3y-z=1 \\ x-y-z=-1 \end{cases}$

Solución:

a) $\left.\begin{array}{r} 3x+2y+z=7 \\ 2x-2y-z=8 \\ x+5y+z=-2 \end{array}\right\} \boxed{\begin{array}{l} 1^a \\ 2^a+1^a \\ 3^a-1^a \end{array}} \rightarrow \left.\begin{array}{r} 3x+2y+z=7 \\ 5x=15 \\ -2x+3y=-9 \end{array}\right\} \rightarrow \left.\begin{array}{r} x=3 \\ y=\frac{-9+2x}{3}=-1 \\ z=7-3x-2y=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=3 \\ y=-1 \\ z=0 \end{array}}$

b) $\left.\begin{aligned} 3x+y-2z&=6\\ 2x-y+3z&=-8\\ x+y-z&=4 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a+1^a\\ 3^a-1^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} 3x+y-2z&=6\\ 5x+z&=-2\\ -2x+z&=-2 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a\\ 3^a-2^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} 3x+y-2z&=6\\ 5x+z&=-2\\ -7x&=0 \end{aligned}\right\}$ $\rightarrow$

$\rightarrow$ $\left.\begin{aligned} x&=0\\ z&=-2-5x=-2\\ y&=6-3x+2z=2 \end{aligned}\right\}$ $\boxed{\begin{matrix}x=0\\ y=2\\ z=-2\end{matrix}}$

c) $\left.\begin{aligned} -2x-y+z&=-4\\ 3x+y-2z&=6\\ 2x+y+z&=6 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a+1^a\\ 3^a+1^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} -2x+-y+z&=-4\\ x-z&=2\\ 2z&=2 \end{aligned}\right\}$ $\left.\begin{aligned} z&=1\\ x&=2+z=3\\ y&=-2x+z+4=-1 \end{aligned}\right\}$ *Solución :* $x=3$, $y=-1$, $z=1$

d) $\left.\begin{aligned} 2x-y+2z&=2\\ x+2y-z&=3\\ 2x-y+3z&=1 \end{aligned}\right\}$ $\boxed{\begin{matrix}2^a\\ 1^a\\ 3^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} x+2y-z&=3\\ 2x-y+2z&=2\\ 2x-y+3z&=1 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-2\cdot 1^a\\ 3^a-2\cdot 1^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} x+2y-z&=2\\ -5y+4z&=-4\\ -5y+5z&=-5 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-3^a\\ 3^a:(-5)\end{matrix}}$ $\rightarrow$

$\rightarrow$ $\left.\begin{aligned} x+2y-z&=3\\ -z&=1\\ y-z&=1 \end{aligned}\right\}$ $\rightarrow$ $\left.\begin{aligned} z&=-1\\ y&=1+z=0\\ x&=3-2y+z=2 \end{aligned}\right\}$ $\boxed{\begin{matrix}x=2\\ y=0\\ z=-1\end{matrix}}$

e) $\left.\begin{aligned} x+2y-2z&=6\\ x-3y+z&=-7\\ 2x-y+z&=-3 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-1^a\\ 3^a-2\cdot 1^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} x+2y-2z&=6\\ -5y+3z&=-13\\ -5y+5z&=-15 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-3^a\\ 3^a:(-5)\end{matrix}}$ $\rightarrow$

$\rightarrow$ $\left.\begin{aligned} x+2y-2z&=6\\ -2z&=2\\ y-z&=3 \end{aligned}\right\}$ $\rightarrow$ $\left.\begin{aligned} z&=\frac{2}{-2}=-1\\ y&=3+z=3-1=2\\ x&=6-2y+2z=6-4-2=0 \end{aligned}\right\}$ *Solución* : $x=0$, $y=2$, $z=-1$

f) $\left.\begin{aligned} x+y-z&=2\\ 2x-2y+3z&=1\\ x+2y-z&=4 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-2\cdot 1^a\\ 3^a-1^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} x+y-z&=2\\ -4y+5z&=-3\\ y&=2 \end{aligned}\right\}$ $\rightarrow$ $\begin{aligned} &y=2\\ &z=\frac{-3+4y}{5}=\frac{-3+8}{5}=1\\ &x=2-y+z=2-2+1=1 \end{aligned}$

Solución : $x=1$, $y=2$, $z=1$

g) $\left.\begin{aligned} x-2y+z&=6\\ 3x+y-z&=7\\ x-y+2z&=6 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-3\cdot 1^a\\ 3^a-1^a\end{matrix}}$ $\rightarrow$ $\left.\begin{aligned} x-2y+z&=6\\ 7y-4z&=-11\\ y+z&=0 \end{aligned}\right\}$ $\boxed{\begin{matrix}1^a\\ 2^a-7\cdot 3^a\\ 3^a\end{matrix}}$ $\rightarrow$

h) $\left.\begin{array}{r} x-y+2z=7 \\ x+y-3z=-5 \\ 2x-y+2z=9 \end{array}\right\}$ $\boxed{\begin{array}{l} 1^a \\ 2^a-1^a \\ 3^a-2\cdot 1^a \end{array}}$ $\rightarrow$ $\left.\begin{array}{r} x-y+2z=7 \\ 2y-5z=-12 \\ y-2z=-5 \end{array}\right\}$ $\boxed{\begin{array}{l} 1^a \\ 2^a-2\cdot 3^a \\ 3^a \end{array}}$ $\rightarrow$

$\rightarrow$ $\left.\begin{array}{r} x-y+2z=7 \\ -z=-2 \\ y-2z=-5 \end{array}\right\}$ $\rightarrow$ $\left.\begin{array}{r} z=2 \\ y=-5+2z=-5+4=-1 \\ x=7+y-2z=7-1-4=2 \end{array}\right\}$ *Solución* : $x=2$, $y=-1$, $z=2$

i) $\left.\begin{array}{r} x+y+2z=6 \\ x-3y-z=1 \\ x-y-z=-1 \end{array}\right\}$ $\boxed{\begin{array}{l} 1^a \\ 2^a-1^a \\ 3^a-1^a \end{array}}$ $\rightarrow$ $\left.\begin{array}{r} x+y+2z=6 \\ -4y-3z=-5 \\ -2y-3z=-7 \end{array}\right\}$ $\boxed{\begin{array}{l} 1^a \\ 2^a-2\cdot 3^a \\ 3^a \end{array}}$ $\rightarrow$

$\rightarrow$ $\left.\begin{array}{r} x+y+2z=6 \\ 3z=9 \\ -2y-3z=-7 \end{array}\right\}$ $\rightarrow$ $\left.\begin{array}{r} z=\frac{9}{3}=3 \\ y=\frac{-7+3z}{-2}=\frac{-7+9}{-2}=-1 \\ x=6-y-2z=6+1-6=1 \end{array}\right\}$ *Solución* : $x=1$, $y=-1$, $z=3$